A robust underwater visibility parameter.

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Abstract

We review theoretical models to show that contrast reduction at a specific wavelength in the horizontal direction depends directly on the beam attenuation coefficient at that wavelength. If a black target is used, the inherent contrast is always negative unity, so that the visibility of a black target in the horizontal direction depends on a single parameter only. That is not the case for any other target or viewing arrangement. We thus propose the horizontal visibility of a black target to be the standard for underwater visibility. We show that the appropriate attenuation coefficient can readily be measured with existing simple instrumentation. Diver visibility depends on the the photopic beam attenuation coefficient, which is the attenuation of the natural light spectrum convolved with the spectral responsivity of the human eye (photopic response function). In practice, it is more common to measure the beam attenuation coefficient at one or more wavelength bands. We show that the relationship: visibility is equal to 4.8 divided by the photopic beam attenuation coefficient; originally derived by Davies-Colley (1988), is accurate with an average error of less than 10% in a wide variety of coastal and inland waters and for a wide variety of viewing conditions. We also show that the beam attenuation coefficient measured at 532 nm, or attenuation measured by a WET Labs commercial 20 nm FWHM transmissometer with a peak at 528nm are adequate substitutes for the photopic beam attenuation coefficient, with minor adjustments.

Introduction

Special Operations and Mine Warfare require the prediction of visibility for divers and cameras using ambient (natural) light.

Visibility is a rather poorly defined concept that can mean many things. It ultimately predicts the ability of some observer (human or instrumental) to detect some object in a given environment. The resolution of the visibility problem can thus be very complex. In general it can be stated that if all the following are known: 1) all the characteristics of the target (size, shape, spectral reflectivity, markings, etc.) ; 2) the optics of the detector; 3) the inherent optical properties (spectral directional light scattering, absorption, and fluorescence characteristics) of the medium and the complete reflectance characteristics of the bottom; and 4) the external lighting conditions of the medium, one can, by use of classical radiative transfer, combined with Fourier optics, predict with great accuracy what a given object will look like in a given detector system at a given distance. The number of parameters that enter in such calculations however, are very large, far too large to be of use in operational diver situations. While, in theory, solutions to such problems can be obtained, the large number of parameters involved, of which a number must be guessed, guarantee that the solution will not likely be close to reality (see Table 1 in the discussion section). Such a complete solution is thus not appropriate for conflict situations, where quick deployment decisions must be made.

What is needed is a simple but accurate approach to visibility of objects that would include mines and divers. Visibility of these objects in typical situations is not limited by the angle they subtend. For such objects in ambient lighting conditions it is appropriate to look at the contrast reduction as a means to describe visibility. That is the approach taken in this paper. Once one bypasses Fourier optics (visibility of objects based on modulation transfer functions), however, one can no longer expect to be able to predict visibility of small features, such as letters, numbers, or other small scale details on objects. Only the detectibility of the objects themselves is analyzed. We will show below that the horizontal visibility of a black target meets all the requirements for a robust underwater visibility parameter.

Background

Starting in World War II and continuing until the mid 1970's the U.S.Navy extensively funded research in visibility. This effort laid the ground work for much of Ocean Optics as we know it today. The work of Preisendorfer, Duntley, Tyler, and Petzold are legendary and are still frequently used today. Their visibility equations were derived directly from the equation of radiative transfer and are presented below. In the review below we often reference Preisendorfer (1976), Duntley (1963), and the review by Jerlov(1976). The references summarize much of the decades long theoretical and experimental work at the Visibility Laboratory of the Scripps Institute of Oceanography. We use the IAPSO nomenclature of Jerlov(1976) in this paper.

A fundamental law of visibility as derived by Duntley (1963), Jerlov (1976), and Preisendorfer (1976), is that the difference of the target and background radiances at a given wavelength attenuates as e^{-cr}, where c is the beam attenuation coefficient at that wavelength, and r is the range from the observer to the target. The derivation of this law follows directly from the equation of radiative transfer, and is presented below:

The equation of radiative transfer for a plane parallel medium without internal sources at a specific wavelength is given by:

$$\cos(\theta)\frac{dL(\theta,\phi,z)}{dz} = - c(z)\, L(\theta,\phi,z) + \int_0^{2\pi}\int_0^{\pi} \beta(\theta,\phi,\theta',\phi',z)L(\theta',\phi',z)\, \sin\theta'\, d\theta'\, d\phi' \qquad (1)$$

we can replace the last term by $L^*(\theta,\phi,z)$. This is the so-called path function.
We then get:

$$\cos(\theta)\frac{dL(\theta,\phi,z)}{dz} = - c(z)\, L(\theta,\phi,z) + L^*(\theta,\phi,z). \qquad (2)$$

For the target radiance $L_T(\theta,\phi,z)$ and the adjacent background radiance $L_B(\theta,\phi,z)$ we can write the following equations, assuming c is a constant and the path functions for the adjacent target and background radiances are the same:

$$\cos(\theta)\frac{dL_T(\theta,\phi,z)}{dz} = - c\, L_T(\theta,\phi,z) + L^*(\theta,\phi,z). \qquad (3)$$

$$\cos(\theta)\frac{dL_B(\theta,\phi,z)}{dz} = - c\, L_B(\theta,\phi,z) + L^*(\theta,\phi,z). \qquad (4)$$

We take the difference of Eqs 3 and 4:

$$\cos(\theta)\frac{d[L_T(\theta,\phi,z)- L_B(\theta,\phi,z)]}{dz} = - c\, [\, L_T(\theta,\phi,z) - L_B(\theta,\phi,z)]\, . \qquad (5)$$

Integration of Eq.5 along a line of sight from r'=0 at the target to r'=r at the observer gives (note that $r = z/\cos\theta$ and that $dr = dz/\cos\theta$):

$$[\, L_{Tr}(\theta,\phi,z) - L_{Br}(\theta,\phi,z)] = [\, L_{T0}(\theta,\phi,z_T) - L_{B0}(\theta,\phi,z_T)]\, \exp(- cr). \qquad (6)$$

The equation of radiative transfer thus shows that the difference of the target and background radiances attenuates as e^{-cr}. This result was obtained by Duntley (1963), Jerlov (1976), and Preisendorfer (1976).

The Visibility Laboratory contrast model

The contrast used by Preisendorfer, Duntley, and Jerlov is the visibility contrast defined by

$$C_v = \frac{L_T(\theta,\phi,z) - L_B(\theta,\phi,z)}{L_B(\theta,\phi,z)} . \quad (7)$$

A combination of Eqs.7 and 6 shows that we may write (Jerlov,1976, Preisendorfer, 1976):

$$\frac{C_{vr}(\theta,\phi,z)}{C_{v0}(\theta,\phi,z_T)} = \exp(-cr) \frac{L_{B0}(\theta,\phi,z_T)}{L_{Br}(\theta,\phi,z)} . \quad (8)$$

The background radiance has an attenuation coefficient defined by:

$$K_B(\theta,\phi,z) = \frac{-1}{L_B(\theta,\phi,z)} \frac{dL_B(\theta,\phi,z)}{dz} , \quad (9)$$

so that $\frac{L_{B0}(\theta,\phi,z_T)}{L_{Br}(\theta,\phi,z)} = \exp\,[K_B(\theta,\phi,z)z] = \exp\,[K_B(\theta,\phi,z)\ r\cos\theta]$, and (10)

$$\frac{C_{vr}(\theta,\phi,z)}{C_{v0}(\theta,\phi,z_T)} = \exp[-cr + K_B(\theta,\phi,z)\ r\cos\theta] \quad (11)$$

Eq.11 was derived by Preisendorfer(1976), Duntley(1963), and Jerlov(1976).

Eq. 11 is the fundamental contrast model of Preisendorfer(1976), Duntley(1963), and Jerlov(1976). It was extensively tested by Duntley and Tyler, and was found to be remarkably robust based on very extensive experimental work. Note that the experimental work did not include use of monochromatic beam attenuation measurement but rather of a relatively broad band green light source meter. The effects of the wavelength dependence of the attenuation measurement will be discussed in detail below. Suffice it to say for now that Eq. 11 was tested extensively and found to be applicable within reasonable bounds when using the broad band green attenuation meters.

Duntley (1963) sums up decades of underwater visibility experiments by stating:

" Along an underwater path of sight a remarkable proportion of the objects ordinarily encountered can be seen at limiting ranges between 4 and 5 times the distance $1/[c - K(\theta,\phi,z)\ \cos\theta]$, regardless of their size or the background against which they appear, providing ample daylight prevails."

Similar results were found by Lythgoe (1971) and Davies-Colley (1988) who showed an excellent relationship between horizontal sighting range of a black 200 mm diameter disk and c, where c was measured with a white light source transmissometer equipped with a Wratten #61 green filter, which approximates the photopic (human eye sensitivity) response function. The slope Ψ of the visibility range versus photopic c was found to be 4.8 with little curvature in the relationship. The small dependence of Ψ on c was also determined by Davies-Colley. When the two linear relationships were taken into account it was found that c could be determined from the visibility range to an accuracy of 8%. Inverting this relationship would thus seem to indicate that the visibility range in the horizontal direction can be predicted from c to an accuracy of better than 10 %. This would seem to be more than sufficient for operational situations.

We should take the extensive theoretical and experimental work of Preisendorfer, Duntley, and Tyler into account when designing a "simple" visibility meter. All contrast reduction depends to first order on c, and only on c in the horizontal direction. To second order, depending on direction, $K(\theta,\phi,z) \cos\theta$, plays a role. Near the surface $K(\theta,\phi,z)$ depends strongly on direction, but at greater depths can be approximated by asymptotic K, the irradiance attenuation coefficient. Note that for looking vertically down, we get the well known Secchi depth dependence on c + K.

Davies-Colley (1987) measured the visibility range (y) of a black target versus the green light beam attenuation coefficient for 17 data points from lakes and rivers in New Zealand. He found a relationship given by: visibility range $y = \Psi/c$. The average value of Ψ was 4.8, with a small linear dependence on c.

There is thus ample historic evidence to suggest that the horizontal visibility of a black target, y, is governed by a simple law $y = 4.8/c$, where c is a green light attenuation measurement. This parameter would thus be an ideal underwater visibility standard.

Photopic versus monochromatic attenuation coefficients.

Eq. 11 was strictly valid only for monochromatic light, yet when used by experimentalists it was found to apply to broad band green light sources as well. This observation requires further analysis. The eye perceives photopic parameters, that is it observes light spectra convolved with the spectral sensitivity of the human eye. Photopic quantities are described in various tomes on Photometry (see for example Mobley, 1994, chapter 2). Suffice it to say here that we wish Eq. 11 to hold for photopic quantities.

Rewriting Eq. 11for photopic quantities yields:

$$C_{vr} = \frac{N_T(r) - N_B(r)}{N_B(r)} = -\exp[-\alpha\, r] \qquad (12)$$

We have simplified the notation, so that $N_T(r)$ is the photopic radiance of the target a horizontal distance r from the target, and $N_B(r)$ is the photopic radiance of the background. α is the attenuation coefficient of the image forming light. This notation is the original Vis. Lab. notation and seems appropriate here. All parameters in Eq. 12 refer to photopic quantities. What is the meaning of contrast for a photopic receiver such as the human eye? What is the meaning of a photopic beam attenuation coefficient?

Eq. 12 can be rewritten as:

$$N_T(r) = N_B\,[1-\exp(-\alpha\, r)] \qquad (13)$$

The background radiance is not a function of r as it can be assumed to be constant at a given depth in a given direction.

Eq. 13 is postulated to be true for the image forming light, but it is true for monochromatic light as we saw in Eq. 11 :

$$L_T(\lambda,r) = L_B(\lambda)\,[1-\exp(-c(\lambda)r)] \qquad (14)$$

We now need to reconcile Eqs. 11 and 13.

From the definition of luminance (Mobley, 1994) we find that:

$$N_T(r) = K_m \int_{400}^{700} L_T(r,\lambda)Y(\lambda)\,d\lambda \qquad (15)$$

and similarly for $N_B(r)$. $Y(\lambda)$ is the photopic luminosity function, which describes the relative sensation of brightness perceived by the human eye, when illuminated by light with the same radiance, but at different wavelengths. K_m is the maximum luminous efficacy. Substitution into Eq. 14 yields:

$$N_T(r) = K_m \int_{400}^{700} L_T(r,\lambda)Y(\lambda)\,d\lambda = K_m \int_{400}^{700} L_B(\lambda)Y(\lambda)\,[1-\exp(-c(\lambda)r)]d\lambda \qquad (16)$$

Let us assume that the background radiance is uniform with wavelength (Duntley specified "ample daylight") and set its photopic luminosity equal to one (units of lumen):

$$N_B(r) = 1 = K_m \int_{400}^{700} L_B(r,\lambda)Y(\lambda)\,d\lambda \ ; \text{ so that all } L_B(\lambda) = \left[K_m \int_{400}^{700} Y(\lambda)\,d\lambda \right]^{-1} \qquad (17)$$

We now combine Eqs. 13 and 16 for the uniform radiance of Eq. 17:

$$N_T(r) = [1-\exp(-\alpha\, r)] = K_m \int_{400}^{700} L_B(\lambda)Y(\lambda)\,[1-\exp(-c(\lambda)r)]d\lambda \ ;$$

$$[1-\exp(-\alpha\, r)] = K_m\left[K_m \int_{400}^{700} Y(\lambda)\,d\lambda \right]^{-1} \int_{400}^{700} Y(\lambda)\,[1-\exp(-c(\lambda)r)]d\lambda \ ;$$

$$[1-\exp(-\alpha\, r)] = 1 - \left\{ \int_{400}^{700} Y(\lambda)\,[\exp(-c(\lambda)r)]d\lambda \right\}\left[\int_{400}^{700} Y(\lambda)\,d\lambda \right]^{-1} \qquad (18)$$

$$\exp(-\alpha\, r) = \left\{ \int_{400}^{700} Y(\lambda)\,[\exp(-c(\lambda)r)]d\lambda \right\}\left[\int_{400}^{700} Y(\lambda)\,d\lambda \right]^{-1}$$

$$\alpha = -(1/r)\ln\left\{ \int_{400}^{700} Y(\lambda)\,[\exp(-c(\lambda)r)]\,d\lambda \;/\; \left[\int_{400}^{700} Y(\lambda)\,d\lambda \right] \right\} \qquad (19)$$

Eq. 19 provides the connection between photopic α and monochromatic $c(\lambda)$. If we now substitute $Y_n(\lambda)$ for the normalized photopic luminosity function $Y(\lambda)$,

$Y_n(\lambda) = Y(\lambda)/\left[\int_{400}^{700} Y(\lambda)\,d\lambda \right] = Y(\lambda)/(106.7)$, we get:

$$\alpha = -(1/r)\ln\left\{ \int_{400}^{700} Y_n(\lambda)\,[\exp(-c(\lambda)r)]\,d\lambda \right\} \qquad (20)$$

If the photopic attenuation coefficient is defined as in Eq. 20, and if the prevailing light is spectrally flat, Eqs. 11 and 12 are equivalent. It should be noted that for a spectral coefficient such as in Eq. 20, Beer's law does not strictly apply. Nor does it for any non-monochromatic attenuation coefficient.

Eq. 20, obtained in a somewhat roundabout method from visibility arguments, also shows how to construct a photopic α-meter: One puts a photopic filter in front of a spectrally flat white light source and measures the attenuation. It is for this reason that Duntley, Preisendorfer, and Davies-Colley used white light sources with Wratten #61 filters, which approximate $Y(\lambda)$.

Modeling photopic versus monochromatic beam attenuation.

In general $c(\lambda) = c_w(\lambda) + c_p(\lambda) + c_g(\lambda)$ i.e. the total attenuation coefficient at a wavelength is the sum of the attenuation coefficients of water, particles, and yellow matter at that wavelength. Twardowski et al. (2001) have discussed the spectral shape of the particulate beam attenuation coefficient. They conclude that $c_p(\lambda)$ being proportional to $\lambda^{-\gamma}$ is a good model. Here we will use :

$$c_p(\lambda)/c_p(532) = (\lambda/532)^{-\gamma}. \qquad (21)$$

γ has values that typically range from 0 to 1.5 . We will ignore absorption by yellow matter as its absorption is weak where $Y_n(\lambda)$ is large. The attenuation of water as determined by Pope and Frye (1997) is used here.

Eq. 20 now contains three independent parameters, $c_p(532)$, γ , and r. Since we are interested in the visibility range, we will set $r = 4.8/\alpha$, so that we can explore the dependence of α on $c_p(532)$ and γ.

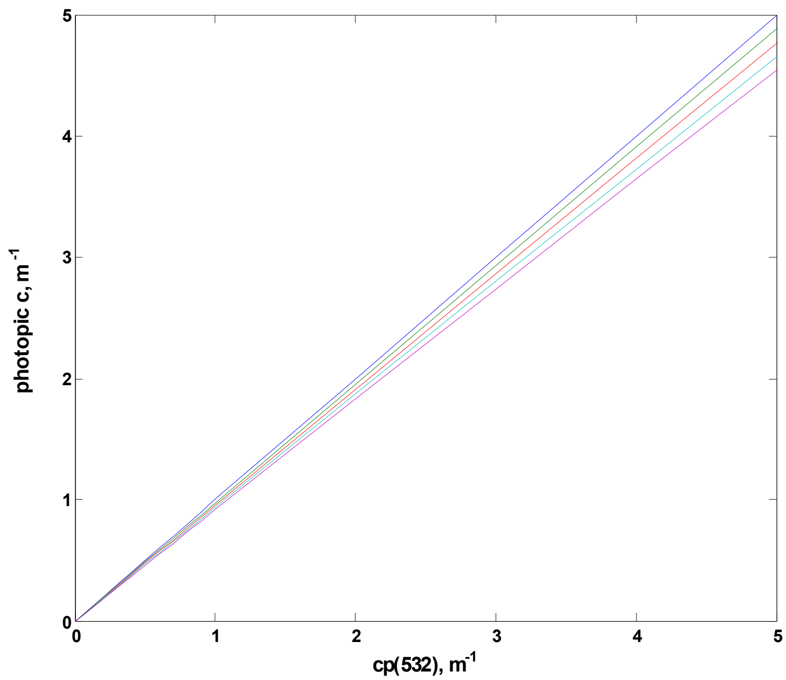

Figure 1. Photopic c versus monochromatic $c_p(532)$ as a function of $\gamma = 0, 0.5, 1.0, 1.5, 2.0$, from upper to lower line, respectively.

Note that for $\gamma = 0$, all c-meters at all wavelengths, calibrated against pure water should read the same, as the attenuation spectrum is flat, hence the 1:1 correlation for $\gamma = 0$.

Based on the model results presented in Fig. 1, we may conclude that an error of less than 10% is made for visibility predictions when using c(532) rather than photopic c.

So far we have used a monochromatic wavelength of 532 nm. It should be noted however, that a commercial instrument such as the WET Labs beam attenuation meter (a nominal 532 nm c-meter) actually has a peak at 528 nm and a FWHM of 20nm. It thus has a much broader spectrum than the monochromatic 532 meter used so far in this analysis. Using these spectral characteristics one can calculate the relationship between photopic and WET Labs green beam attenuation and hence visibility range. It was found that 4.8/[WET Labs green c*0.945 +0.117]correlates to within 5% with the photopic visibility range for all γ. This is shown in Fig. 2.

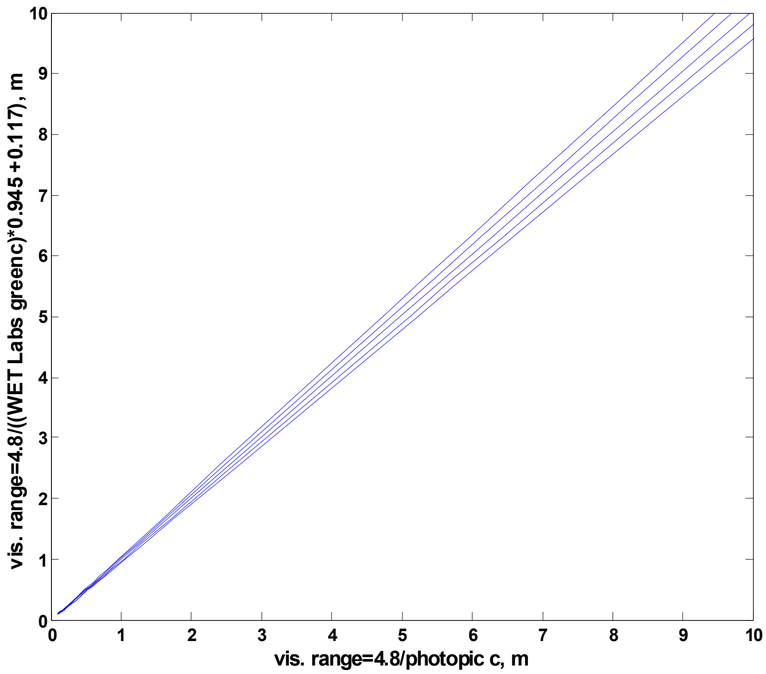

Figure 2. A theoretical comparison of the visibility range obtained with a WET Labs green c-meter with a correction for offset and the photopic visibility range.

The 0.945 factor comes in because the central wavelength of the WET Labs green c meter is less than the central wavelength of the photpic response function. The 0.117 factor is introduced because the photopic visibility range includes attenuation by water, whereas the commercial device is calibrated against pure water, so that pure water attenuation must be added back in.

Since the light source for the WET Labs green c-meter is a non- monochromatic light emitting diode, the fit to photopic beam attenuation is better than for monochromatic c(532).

It is clear from the figure that for practical purposes no correction is needed. It should be noted that variability in human vision, and hence visibility range as determined by various individuals is greater than 10% (Blackwell, 1946).

We thus find that with a minor correction, commercial green attenuation meters are well-suited to predict the photopic visibility range.

Observations

Davies-Colley (1988) showed an excellent relationship between horizontal sighting range of a black 200 mm diameter disk and Ψ/c (blue circles, Fig. 3), where c was measured with a white light source transmissometer equipped with a nearly photopic response filter. Ψ was found to be a weak function of c and averaged 4.8.

Our own data was taken with a WET Labs ac-9 spectral attenuation meter (red circles in Fig. 3). As shown in Fig. 2 the error of using c(532) rather than photopic c should be less than 10%. This is well demonstrated by the data in Fig. 3). Horizontal visibility of a 25 cm diameter black target is well described by 4.8/c(532), as Fig. 3 shows. These data contain a wide variety of locations, such as coastal ocean, estuaries, rivers, and lakes. Similarly, a wide range of illuminations (direct sun, 100% overcast, etc.) are included.

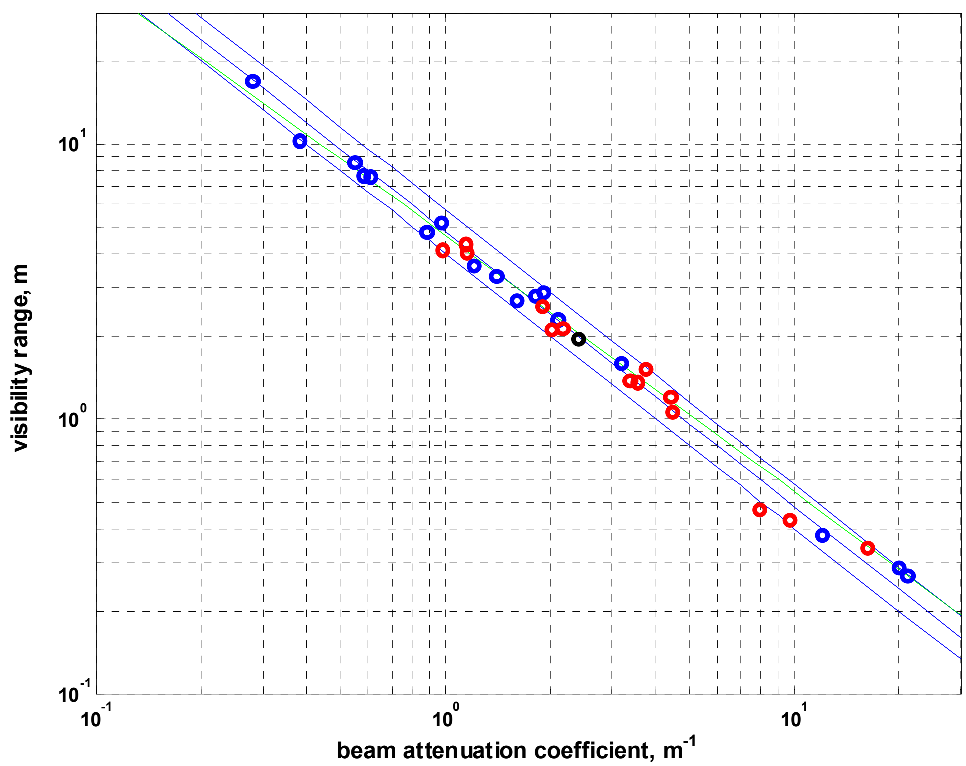

Figure 3. Horizontal visibility of a 25 cm diameter black target. Blue points, Davies-Colley, "green" c-meter; red points, Zaneveld, c(532).; black point, Twardowski.
Blue line vis. range = y = 4.8/c and +/- 20% lines;
green line vis. range= y = (5.207- 0.368 lny)/c . $r^2 = 0.98$.

We also examined the relationship between photopic visibility range and $c_{pg}(650)$ +0.117. (Fig. 4). In this case the errors are slightly larger and the relationship was found to be: Photopic visibility range y = 3.7 /[$c_{pg}(650)$ +0.117].

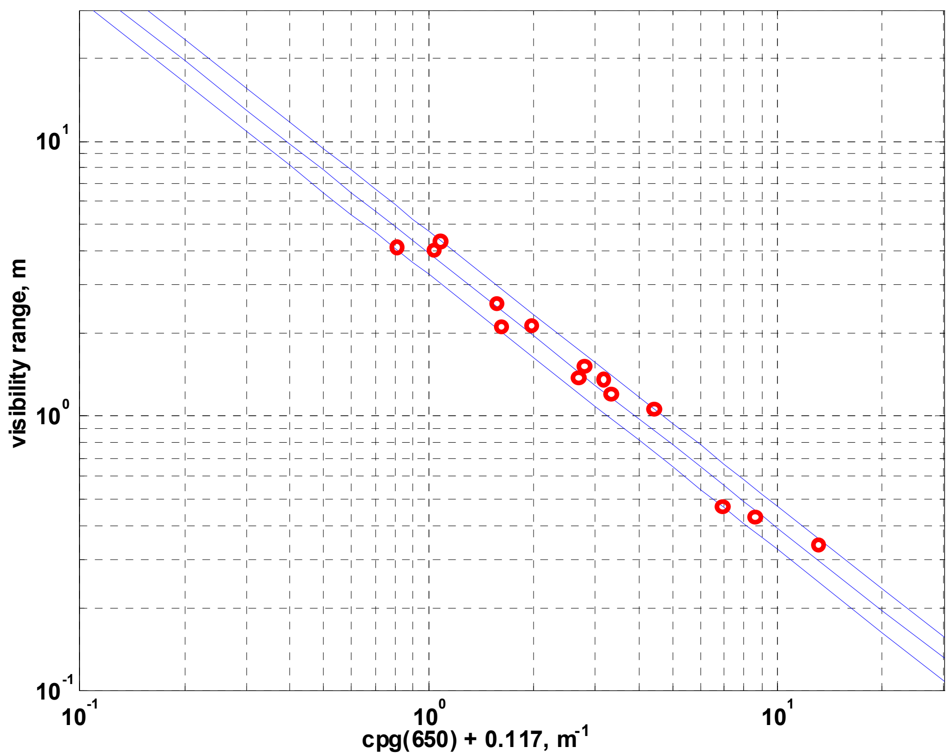

Figure 4. Horizontal visibility of a 25 cm diameter black target. Red points, Zaneveld, $c_{pg}(650)$ +0.117. Blue lines indicate vis. range = y = 3.7/c and +/- 20% lines; $r^2 = 0.97$. Data taken in the same locations as the red circles in Fig.3.

We added the attenuation of pure water at 550 nm, 0.117 m^{-1}, rather than the value for 650 nm, as human vision centers at 550 nm.

Discussion

The theory and the data show that the horizontal visibility of a moderately sized black target (25 cm diameter) is essentially an Inherent Optical Parameter described by $4.8/\alpha$, where α is the photopic beam attenuation coefficient. An excellent proxy for the photopic beam attenuation coefficient is c(532) or the attenuation obtained with a green LED source transmissometer, such as the WET Labs green c-star. The relationship is best when making a slight correction for the wavelength offset and adding the pure water attenuation. Even the attenuation in the red measured with a spectral attenuation meter (WET Labs ac-9), expressed by $c_{pg}(650)$ +0.117, gives an excellent correlation with visibility range.

All other visibility arrangements in terms of target properties, lighting and viewing arrangements require far more parameters and so do not qualify as a simple parameter that is useful to operational divers. Table 1 describes the parameters needed to predict visibility in other than the simple horizontal viewing arrangement of a black target described here.

	Horizontal viewing angle	Vertical viewing angle	Arbitrary viewing angle
Large target; black ; ample daylight	c	c, K	c, K , θ
Large target; arbitrary reflectance; ample daylight	c, ρ, TO(θ, ϕ)	c, K, ρ , TO(θ, ϕ)	c, K, ρ , θ, TD(D),TO(θ, ϕ)
arbitrary size target; black; ample daylight	c, TD(D), C_L(D)	c, K, TD(D),C_L(D)	c, K, C_L(D), θ , TD(D),TO(θ, ϕ)
arbitrary size target; arbitrary reflectance; ample daylight	c, ρ, TD(D), C_L(D)	c, K, ρ , TD(D), C_L(D)	c, K, ρ , C_L(D), θ , TD(D), TO(θ, ϕ)
arbitrary size target; black; arbitrary illumination	c, TD(D), C_L(D), C_L(E)	c, K, TD(D), C_L(D), C_L(E)	c, K, C_L(D), C_L(E) , θ, TD(D), TO(θ, ϕ)
arbitrary size target; arbitrary reflectance; arbitrary illumination	c, ρ, TD(D), C_L(D), C_L(E)	c, K, ρ , TD(D),C_L(D), C_L(E)	c, K, ρ, θ, C_L(D), C_L(E) , TD(D), TO(θ, ϕ)

Table 1. Parameters required to predict visibility range. All situations also require the inherent contrast of the target.

c	spectral beam attenuation coefficient
K	spectral diffuse attenuation coefficient
ρ	spectral target reflectivity
θ	viewing angle
TD(D,S)	target description; diameter and shape
TO(θ, ϕ)	target orientation; zenith angle and azimuth
C_L(D)	threshold contrast as a function of target diameter
C_L(E)	threshold contrast as a function of light level

Bowers (2002) describes calculations and observations of visibility for various sighting situations. If we take his calculated sighting range in the horizontal for a black target (90º and 270º in his figures 2 and 3, and if one were to use Davies-Colley's experimentally determined 4.8/c visibility range rather than the 4/c used by Bowers (i.e. 20% larger) , we find good agreement between the calculated and predicted visibility range (about 10% error on average). Bowers' Fig. 4 refers to a white, round target, for which the 4.8/c rule does not hold as the target has to be black, or nearly so. A shiny round target has as its brightest value the reflected downwelling radiance. The 4.8/c rule is based on the contrast between the horizontal background radiance and a nearly zero inherent target radiance. In other words the contrast at the target needs to be nearly –1. For a shiny sphere, the contrast at the target is likely to be larger than 10 as the downwelling radiance is at least that much larger than the horizontal radiance. The data presented in Bowers (2002) thus support the Davies-Colley (1988) rule.

We conclude that the simple visibility parameter, visibility range = 4.8 / photpic beam attenuation, advocated here is well grounded in theory, is readily measured by proxy apparatus, and is extremely useful to provide divers with a general sense of underwater visibility conditions.

Acknowledgement

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