WET Labs Technical Note

VOLUME SCATTERING FUNCTION FROM A BACKSCATTERING MEASUREMENT AND THE SLOPE OF THE BEAM ATTENUATION SPECTRUM.

The determination of the Fournier-Forand phase function from the backscattering to total scattering ratio and the slope of the beam attenuation spectrum.

Fournier and Forand (1994, FF) have provided a phase function (Volume scattering function normalized to the total scattering coefficient) that approximates the scattering due to an ensemble of particles with a Junge particle size distribution (PSD) and a given average particle index of refraction, n_p. The scattering is approximated by the anomalous diffraction approximation.

Twardowski et al.(2001) have shown that the average or bulk refractive index of particles can be estimated from the hyperbolic slope of the PSD, μ, and the ratio of the particulate backscattering and total scattering coefficients, $B_p=b_{bp}/b_p$. These parameters can be determined from a ECO backscattering sensor and an ac-9. Furthermore, Boss et al. (2001) have shown that the slope of the beam attenuation spectrum can be used to estimate the slope of the PSD. Hence the data from an ac-9 and a backscattering sensor are sufficient input to obtain the FF phase function shape.

For modeling purposes it may be desirable to use a one parameter phase function. The Henyey-Greenstein phase function is often used for this purpose. However it is inappropriate for near forward scattering in ocean waters. The FF is far better suited to modeling of ocean waters. The FF can be turned into a one parameter model by assuming a functional relationship between the hyperbolic slope of the PSD and the bulk particle index of refraction. Mobley et al.(2002) have given such an approximate relationship. Using that relationship, the FF can be used as a one parameter phase function, the only parameter being the backscattering ratio B_p.

Shown below is a MATLAB program that calculates the FF phase function for two situations:
1) the slope of the PSD, μ, and the bulk index of refraction of the particles, n_p are given.
2) The backscattering ratio, $B_p=b_{bp}/b_p$ is given. The program then calculates μ and n_p, and from these the FF phase function. Log- log and log- linear plots of the results are given.

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% Program FournierForand

% This program calculates the Fournier-Forand phase function as described in:
% Mobley et al.(2002) "Phase function effects on oceanic light fields", Applied Optics,
% Vol.41 No.6, pp 1035-1050.

% theta is the angle in degrees, given here in steps of 0.1 degrees.  Hence theta(1)=
% 0 degrees and theta(1801)=180 degrees.
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% The Fournier-Forand normalized (to total scattering) phase function is
% given by betaFF.
% The two parameters in betaFF are nu and delta. nu depends on the slope of the
% hyperbolic particle size distribution,mu. delta depends on the average real index
% of refraction of the particles, np, and theta.

clear;

% There are two solution options.  Either you specify the slope, mu, of the particle size distribution and the
% particulate index of refraction, np. (setup=1),
% or you specify the ratio of particulate backscattering to total scattering Bp, (setup=2). In that case the mu and
% np values are calculated according to the model of Mobley et al. cited above. Note that for Bp=0.5 you get the
% phase function of pure water.  For Bp=0.0183 you get the Petzold average-particle phase function.

% In option 1 the slope of the PSD, mu, can be derived from the slope of the beam attenuation spectrum as obtained
% from an ac-9.  The relationship between the slope of the beam attenuation spectrum, gamma, and mu, was obtained
% Boss et al. (2001), Spectral particulate attenuation and particle size distribution in the bottom boundary layer
% of a continental shelf. J. Geophys Res. 106, 9499-9508 and is used below to get mu from gamma.
% This relationship is: gamma=mu-3+0.5*exp(-2.3*(mu-2.5));

%gamma=1 % gamma must be greater than or equal to 0.02.
%fgamu=inline('-gamma+mu-3+0.5*exp(-2.3*(mu-2.5))','mu','gamma');
%[mufromgamma,fval]=fzero(fgamu,3.5,[],gamma);
%mufromgamma % Take this value and plug it into muin below.

%INPUTS
setup=2; %specify according to the above, which solution you want.
muin=4.0; % specify mu, the slope of the particle size distribution; This must be >3 and <5; Set=0 if setup=2.
npin=1.35; % specify average particle index of refraction. Set=0 if setup=2.
Bpin=0.03; % provide the normalized particulate backscattering.Set=0 if setup=1.

%CALCULATION OF THE FOURNIER-FORAND PHASE FUNCTION
if setup==1;
   mu=muin
   np=npin
else
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%Find a minimum for the two equations below to get nu (See Mobley et al. for details);
%delta90=(0.6777*(0.01-0.3084*nu)^-2);
%Bp=1-(1-delta90^(nu+1)-0.5(1-delta90^nu))/((1-delta90)*delta90^nu);
Bp=Bpin
fff1='-Bp+1-(1-(0.6777*(0.01-0.3084*nu)^-2)^(nu+1)-0.5*(1-(0.6777*(0.01-
0.3084*nu)^-2)^nu))';
fff2='/((1-(0.6777*(0.01-0.3084*nu)^-2))*(0.6777*(0.01-0.3084*nu)^-2)^nu)';
fbpnui=inline([fff1 fff2],'nu','Bp');% Break it into two lines.
[nusol,fval]=fzero(fbpnui,0,[],Bp);% The fval is in there only to distinguish fzero from
the old fzero;
mu=3-2*nusol
np=1.01+0.1542*(mu-3) % This is the relation between mu and np given in Mobley et al.
end

nu=(3-mu)/2;
delta180=(4/(3*(np-1)^2));% delta evaluated at 180 degrees.
for ii=1:1801;
   theta(ii)=(ii-1)/10;%scattering angle in degrees.
   thetar=((ii-1)/10)*pi/180; %convert to radians.
   delta=(4/(3*(np-1)^2))*sin(thetar/2)^2;
   omd=1-delta;
   betaFF(ii)=(1/(4*pi*omd^2*delta^nu))*(nu*omd-(1-delta^nu)+...
      (delta*(1-delta^nu)-nu*omd)*sin(thetar/2)^-2)+...
      ((1-delta180^nu)/(16*pi*(delta180-1)*delta180^nu))*(3*cos(thetar)^2-1);
end

ii=[2:1801];figure(1);loglog(theta,betaFF);
xlabel('scattering angle, degrees');ylabel('Fournier-Forand phase function, sr ^-^1');

ii=[2:1801];figure(2);semilogy(theta,betaFF);
xlabel('scattering angle, degrees');ylabel('Fournier-Forand phase function, sr ^-^1');
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References

Boss et al. (2001), Spectral particulate attenuation and particle size distribution in the bottom boundary layer of a continental shelf. *J. Geophys Res.* **106**, 9499-9508.

Fournier and Forand (1994) “Analytic phase function for ocean water” in Ocean Optics XII, Proc. SPIE, **2558**, 194-201.

Mobley et al.(2002) "Phase function effects on oceanic light fields", Applied Optics, Vol.**41** No.6, pp 1035-1050.

Twardowski et al. (2001) “ A model for estimating bulk refractive index from the optical backscattering ratio and the implications for understanding particle composition in caseI waters,” J. Geophys. Res. **106**(C7) 14129-14142.